A novel model for solar radiation prediction

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Abstract

Energy for fulfilling basic community/individual needs has come to constitute the first article of expectation in all contemporary societies. The exploitation of renewables notably solar in electricity generation has brought relief to the fulfilment of energy demand especially among susceptible communities. In this paper yearly minimum solar radiation of Kano (12.05°N; 08.2°E; altitude 472.5 m; 3 air density 1.1705 kg/m³) for 46 years is used to generate a prediction model that fits the data using autoregressive moving average (ARMA) and a new model termed autoregressive moving average process (ARMAP). Comparison between the ARMA and ARMAP models showed a tremendous improve in the sum of square error reduction between the actual data and the forecasted data by 47%.

Keywords: mean absolute percentage error, solar radiation, sum of square error, root mean square error, time series

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1. Introduction

Although electrical energy is seldom available as a primary form, it is most commonly used in modern living. This is because it is easy to transport between distant locations and can efficiently be used to provide needed power to perform various functions that make existence bearable [1]. Energy demand in the World and in Nigeria has experienced a tremendous increase as human activities are made more convenient through the use of electronics and electrical devices (Computer, phones, electric kettle, washing machine and so on). This demand is projected to triple by the year 2050 [2]. Likely acerbating the already serious greenhouse effect attributed to the massive consumption of energy provided by fossil fuel in the last century [3]. Consequently, a serious reduction in fossil-fuel power sources must occur implying that renewable energy must become dominant [4, 5].

Nigeria has a huge solar energy potential ranging between 1,800 KWh/m² to the Southern region and more than 2,300 KWh/m² to the Northern region as shown in Figure 1 [6, 7]. The abundance of solar energy in Nigeria is huge, the radiated sun’s energy is about 3.8x10²³ KW which is 1.082 million tonnes of oil equivalent (mtoe). This solar potential is more than 4000 times and 13000 times of Nigeria daily production of crude oil and natural gas respectively [8, 9].

There is need to harness renewable energy (RE) sources such as solar energy in developing countries like Nigeria for sustainable energy provision to bring about economic development, rural development, ease of living and provide clean energy from the perspective of the Kyoto agreement. It is imperative to note that only about 40% of the total population in Nigeria have access to electricity, out of which only about 5% of the rural areas are privy to such electricity [10, 11]. The communities with the access hardly enjoy more than 48 hours per week of uninterruptible power supply due to so many problems from the generation down to distribution [12]. Regarding these, Microgrid (especially solar) might be used to mitigate the problem [8]. Solar photovoltaic technology has numerous advantages, since it reduces greenhouse effect [13, 14], can be installed in a modular, quickly increase generation even to remotest community and can easily be funded by individuals or cooperatives.
Solar energy conversion requires the knowledge of the solar radiation data of the area of application as it is not globally uniform [1-15]. The availability of the empirical data makes it feasible to create a model using various techniques. However, if the data is lacking then the use of empirical equations of areas with almost similar climate can be deployed [16]. Electric power generation in Nigeria faces numerous challenges and the increase in the price of oil in the mid-1970s that accompanied prosperity triggered a phenomenal of growth in electrical appliances for social, commercial and industrial use [8]. While such increase in demand occurred almost immediately, the power to operate them could not keep pace. The result was a power grid with performance progressively degenerated and has resisted all attempts to reverse the trend.

The stochastic nature of renewables like solar radiation is its major drawback as it is difficult to easily formulate a prediction model [17]. However, the use of statistical approach such as time series Box and Jenkins procedure [16] ARMA makes it possible to estimate and forecast the next generation of solar radiation data [18]. One of the shortcomings of prediction with ARMA is that it requires many data points for better accuracy and the data must be stationary [1]. The mentioned shortcomings of ARMA motivated the development of a novel hybrid time series model termed ARMAP.

![Figure 1. Map of Nigeria solar energy potential](image)

2. ARMA Model

The solar radiation time series is composed of a sequence of daily solar insolation over the period of 46 years between 1971 and 2016 observations \( \{y_t\} \). A commonly used avenue for time series prediction is the autoregressive moving average (ARMA) models after ensuring that the time series is stationary [19]. ARMA has the capability of extracting useful statistical properties using the well known Box and Jenkins model [20]. The general Autoregressive Moving Average, ARMA \((p,q)\) for a solar radiation \(y_t\), is given by:

\[
y_t = \sum_{k=1}^{p} a_k y_{t-k} + \sum_{h=1}^{q} \beta_h \varepsilon_{t-h} + \varepsilon_t
\]  

\( (1) \)
where $\alpha$, $\beta$ are coefficients of AR and MA respectively; $\varepsilon$ is a random process with zero mean and constant variance. The Box-Jenkins procedure is based on 3 pillars [20, 21] as shown in Figure 2.

2.1. Model Identification

The Autocorrelation Function (ACF) and Partial Autocorrelation Function (PACF) plots are compared with the theoretical catalog to identify model. A pure autoregressive will have its ACF spikes decaying towards zero and the PACF cuts-off after the $p$ lag(s). While the moving average is opposite of autoregressive while the ARMA model have geometrical decay for both ACF and PACF. The lags $\hat{\rho}(k)$ are defined as:

$$\hat{\rho}(k) = \frac{\sum_{i=1}^{k-1} (y_{t-i} - \mu)(y_{t+k-i} - \mu)}{\sum_{i=1}^{N} (y_{t-i} - \mu)^2} \quad (2)$$

where $k$ is the number of lags and $\mu$ is the mean of the solar radiation data, $y_t$.

The partial autocorrelation function (PACF), $\phi_{kk}$ is a representation of the correlation between the deviations in the data and the linear relation from the intermediate variables in the partial autocorrelation function with $k^{th}$ partial correlation coefficient. The PACF is defined as:

$$\phi_{kk} = \frac{\rho(k) - \sum_{j=1}^{k-1} \alpha_{k-1,j} \hat{\rho}_{k-j}}{1 + \sum_{j=1}^{k-1} \alpha_{k-1,j} \hat{\rho}_{j}} \quad (3)$$

$$\phi_{k,j} = \phi_{k-1,j} - \phi_{kk} \phi_{k-1,k-j} \quad j = 1, 2, ..., k - 1 \quad (4)$$

The spikes in the ACF and PACF give guidance to the nature of the model parameters. An ACF that dies down exponentially and PACF that has spikes in lags, $p$ is said to be purely ARMA($p,0$). While the opposite of AR processes indicates moving average (MA) process. The ARMA($p,q$) has both ACF and PACF spikes decay towards zero. Once the ACF and PACF were identified the model parameters number are speculated and several models are used whereof the most parsimonious model is chosen.

2.2. Parameter Estimation

This involves the selection of parameters ($p,q$) based on the behaviour of the autocorrelation function (ACF) and partial autocorrelation function (PACF). There are various methods of estimates such as Yule-Walker, least square, maximum likelihood and so on. The least-square method is used to find the coefficient(s) of the parameter of an autoregressive model. For AR(1) the least square estimate is given by:

$$\alpha_1 = \frac{\sum_{i=1}^{N} (y_{t-1} - \mu)^2}{\sum_{i=1}^{N} y_{t}^2} \quad (5)$$
Considering AR(n) where n>1, then we have:

\[
\begin{pmatrix}
\alpha_1 \\
\alpha_2 \\
\vdots \\
\alpha_n
\end{pmatrix} = \begin{pmatrix}
a_{11} & a_{12} & \cdots & a_{1m} \\
a_{21} & a_{22} & \cdots & a_{2m} \\
\vdots & \vdots & \ddots & \vdots \\
a_{n1} & \cdots & \cdots & a_{nm}
\end{pmatrix}^{-1} \begin{pmatrix}
b_1 \\
b_2 \\
\vdots \\
b_k
\end{pmatrix}
\]

where \( n = m \)

\[
a_{nn} = \sum_{t=1}^{N}(y_{t-n})^2
\]

for \( n \neq m \)

\[
a_{nm} = a_{mn} = \sum_{t=1}^{N}(y_{t-n} - y_{t-m})^2
\]

\[
b_k = \sum_{t=1}^{N}(y_{t} - y_{t-k})^2
\]

where \( y_t \) is the actual data.

2.3. Model Diagnostic Checking

Several ARMA models were computed using the same time series data, but the most parsimonious (adequate) is chosen as the best for the data. There are various ways of diagnostic checking of ARMA model, but all are based on the residuals. The following goodness of fit test is used to obtain the most adequate model:

- The time plot of the residual behaves like the normal distribution, with approximately zero mean and constant variance.
- Spikes of the plots of residuals ACF and PACF should lie within the 95% confidence interval (CI) bounds. That is:

\[
\text{CI}(\text{bound}) = \pm 1.96/\sqrt{N}
\]

where \( N \) is the sample data size (not less than 25% of the total size of the data size)
- The Akaike Information Criterion: For a time series of length \( n \), the lowest AIC gives the best and most parsimonious model. AIC is defined by:

\[
\text{AIC} = n \times \ln \left( \frac{\sum e^2}{n} \right) + 2(p + q)
\]

where \( e^2 \) is the difference between actual and forecasted value; \( n \) is number of observations and \( p \) and \( q \) are model order [20-22].

3. Proposed Autoregressive Moving Average Process (ARMAP) Model

The time series ARMA model falls under the traditional method of forecasting among the three other classifications of forecasting. The other methods have been new techniques and bottom-up approaches. The hybrid of two methods of forecasting is expected to greatly improve the accuracy of the forecasting model of the solar radiation.

A model termed Autoregressive Moving Average Process (ARMAP) is developed using a hybrid system time series including an integral of a certain number of preceding values. The new model, ARMAP is developed based on the principle of Ziegler-Nichols rules for PID controller tuning, where the parameters of the PID controllers can be selected from the mathematical model or by experiments of the plant to improve its stability (transient and steady-state). The ARMAP is developed to fine tune the ARMA time series model. This is accomplished by introducing an integral variable \( V_n \) which is added as a buffer between a stationary and non-stationary time series. It has a length \( w \), bounded \( w-1<ww/4 \).

The proposed new model is defined as:

\[
Y_t = \sum_{k=1}^{p} \alpha_k y_{t-k} + V_{k+1} y + \sum_{h=1}^{S} \beta_h \varepsilon_{t-h} + \varepsilon_t
\]
the parameter, \( V_n \) is defined in (13).

\[
V_n = \frac{1}{2n} \left[ (y_{t-1} + y_{t-n}) + 2 \sum_{k=2}^{n-1} y_{t-k} \right]
\]

(13)

where \( n \) is integer: 2,3,...; \( t \) is integer: 1,2,... and \( k \) is integer: 2,3,... Also, a series PMA(\( z \)) is included, where the parameter \( z \) is added and defined as:

\[
z = \begin{cases} 
p & p > q \\
q & q > p 
\end{cases}
\]

(14)

where \( p \) and \( q \) are the autoregressive and moving average parameters respectively. The matrices of the coefficients of the ARMAP parameter estimation are found as follows:

\[
a_{nn} = \sum_{t=1}^{N} (Y_{t-n})^2
\]

(15)

for \( n \neq m \);

\[
a_{nm} = a_{mn} = \sum_{t=1}^{N} (Y_{t-n} - V_n)^2
\]

(16)

\[
b_k = \sum_{t=1}^{N} (Y_t - Y_{t-k})^2
\]

(17)

where \( Y_t \) is the solar radiation data.

### 4. Forecasting and Performance of ARMA and ARMAP

The veracity of the developed model is tested by predicting the next generation of solar radiation data using the available data, where MS-Excel is used for all calculations and data fitting. Accuracy is a major fundamental of any forecasting model. While a root mean square error (RMSE) of less than 20% is significant yet the lesser the error the better, even though an over fitting should be avoided so as not to negate the principle of parsimony. Consider \( A(t) \) is the actual data, \( F(t) \) is the forecasted solar radiation data and \( n \) is the data size. The various performance indices of forecasting models considered in this work are:

\[
U = \frac{\left[ \sum_{t=1}^{n} \left( \frac{A(t) - F(t)}{n} \right)^2 \right]^{1/2}}{\left[ \sum_{t=1}^{n} \frac{A(t)^2}{n} \right]^{1/2} \cdot \left[ \sum_{t=1}^{n} \frac{F(t)^2}{n} \right]^{1/2}}
\]

(18)

\[
RMSE = \left( \frac{1}{n} \sqrt{\sum_{t=1}^{n} (A(t) - F(t))^2} \right) \times 100
\]

(19)

\[
MAPE = \frac{1}{n} \sum_{t=1}^{n} \left| \frac{A(t) - F(t)}{A(t)} \right|
\]

(20)

\[
SSE = \sum_{t=1}^{n} (A(t) - F(t))^2
\]

(21)

where \( U \) is Theil’s U-statistic, RMSE is Root Mean Square Error, MAPE is Mean Absolute Percentage Error and SSE is Sum of Square Error [23-25].

### 5. Results and Discussion

#### 5.1. Solar Radiation Data

The 46 years (1\textsuperscript{st} January, 1971 to 31\textsuperscript{st} December, 2016) daily mean solar radiation data (MJ/m\textsuperscript{2}/day) were collected from the Nigerian Metrological Agency (NiMet). Figure 3 shows the monthly average of 46 years data. The maximum solar radiation is recorded in March with an average of 25.15MJ/m\textsuperscript{2}/day and August has the minimum with an average of 19.57MJ/m\textsuperscript{2}/day. The months with the lowest solar radiation coincided with the rainy season as such the influence of cloud cover is imminent. Moreover, the summer season shows the months with the highest values of solar radiation.
5.2. Box and Jenkins ARMA Modelling

Yearly minimum average solar radiation is considered to validate the developed model. Figure 4 shows a time plot of the data. Adding a trendline test the stationarity of the time series found the slope, m, to be -0.0394. However, the first difference of the data showed a slope of m=0.0004 that is almost zero and makes the time series stationary. The Autocorrelation Function (ACF) and partial autocorrelation function (PACF) plots shown in Figures 5 (a) and (b) respectively show that several lags (k) are within 95% confidence interval (CI) bounds ($\pm 1.96/\sqrt{N}$ or $\pm 0.2889$, where $N=46$), hence a stationary time series. The ACF decays in an oscillation form after the second lag and the PACF plot cuts off just after few lags indicating ARMA(2,0) or higher rank.

Least square parameter estimation method is used to compute several ARMA models; including ARMA(1,0), ARMA(1,1), ARMA(2,0), ARMA(2,1), ARMA(3,0) and ARMA(3,1). In Table 1 the AIC and mean square error (MSE) of all the models are presented with ARMA(3,0) having the lowest values of AIC and MSE at 15.2006 and -0.01943. Therefore, ARMA(3,0) is chosen as the most parsimonious model.

The ARMA(3,0) model institutes that the yearly mean solar radiation, $\gamma_t$ depends on $0.4674$, $\alpha_1$ of the value a year before $(\gamma_{t-1})$ subtracted by 0.0466, $\alpha_2$ of the value of 2 years before $\gamma_{t-2}$ added to 0.0820, $\alpha_3$ of the value 3 years before plus a random variable, $\varepsilon_t$ of (-0.0843). The mean, $\mu$ of the random variable is 0.0004 and a standard deviation, $\sigma$ of 1.1194. The time plot, ACF and PACF of the residual error of the model ARMA(3,0) displayed in Figures 6 (a) and (b) has almost all of the lags of the correlations within the CI bounds. The time plot of the residual in Figure 7 is a behaving like a white noise with approximately zero mean ($\mu=0.006$). This point to the suitability of ARMA(3,0) to the data.

The model ARMA(3,0) is used to forecast the solar radiation data and the result in Figure 8 shows a good agreement between the actual data and the forecasted data. The error analysis for all the selected models also show that ARMA(3,0) has the lowest values of MAPE, RMSE and SSE at 0.0422, 0.04002 and 0.0388 as presented in Table 2.
Figure 6. (a) Autocorrelation Function (ACF) plot of residuals of ARMA (3,0) (b) Partial Autocorrelation Function (PACF) plot of residuals of ARMA (3,0)

Figure 7. Time plot of residuals of ARMA (3,0)

Figure 8. Measured Vs ARMA(3,0) of mean yearly minimum solar radiation (1971-2016)

Table 1. AIC and MSE Values of ARMA Models

<table>
<thead>
<tr>
<th>MODEL</th>
<th>AIC</th>
<th>MSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>ARMA(1,0)</td>
<td>18.2776</td>
<td>-0.83726</td>
</tr>
<tr>
<td>ARMA(1,1)</td>
<td>26.9191</td>
<td>-0.80962</td>
</tr>
<tr>
<td>ARMA(2,0)</td>
<td>17.1524</td>
<td>-0.81788</td>
</tr>
<tr>
<td>ARMA(2,1)</td>
<td>26.9191</td>
<td>-0.81788</td>
</tr>
<tr>
<td>ARMA(3,0)</td>
<td>15.2006</td>
<td>-1.9479</td>
</tr>
<tr>
<td>ARMA(3,1)</td>
<td>23.4567</td>
<td>-0.01943</td>
</tr>
</tbody>
</table>

Table 2. RMSE, MAPE, and SSE Values of ARMA Models

<table>
<thead>
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<th>MODEL</th>
<th>RMSE</th>
<th>MAPE</th>
<th>SSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>ARMA(1,0)</td>
<td>0.0182</td>
<td>1.8201</td>
<td>77.5609</td>
</tr>
<tr>
<td>ARMA(1,1)</td>
<td>0.0176</td>
<td>1.7600</td>
<td>77.5762</td>
</tr>
<tr>
<td>ARMA(2,0)</td>
<td>0.0178</td>
<td>1.7780</td>
<td>77.5715</td>
</tr>
<tr>
<td>ARMA(2,1)</td>
<td>0.0423</td>
<td>4.2347</td>
<td>77.5559</td>
</tr>
<tr>
<td>ARMA(3,0)</td>
<td>0.0004</td>
<td>0.0422</td>
<td>0.03885</td>
</tr>
<tr>
<td>ARMA(3,1)</td>
<td>0.0399</td>
<td>3.9882</td>
<td>69.9896</td>
</tr>
</tbody>
</table>

5.3. ARMAP Modelling

The new ARMAP model is developed to eliminate the stationarity bottleneck such as over differencing, stationarity test and so on. Various values of $V_n$ (where $2<n<p+1$) parameter in (14) were selected and the ARMA and PMA methodology is followed to find the best model. Once values of $V_n$ ($V_2$, $V_3$ and $V_4$) are tabulated then it can be used to estimate parameters of the ARMA using different methods such as least square estimation, maximum likelihood, Yule-Walker estimate etc. The model is defined as $\text{ARMA}(p,q)+\text{PMA}(n)$. The sum of square error (SSE) of various PMA(n) increases as n is greater or less than p or q as shown in Figure 9, where for ARMA(3,0), PMA(2) has SSE of 93.9518 that is greater than corresponding PMA(3) with $4.17 \times 10^{-23}$. However, SSE value of PMA(3) is less than that of PMA(4) with $6.14 \times 10^{-21}$. Based on the rule of parsimony ARMA(3,0)+PMA(3) is selected as adequate with a better fit of 48% than ARMA(3,0). The plot of forecasted against the actual data is shown in Figure 10 where the correlation is almost 100%. The SSE, RMSE and MAPE for the model are $1.8708 \times 10^{-21}$, $1.0444 \times 10^{-10}$ and $6.6292 \times 10^{-12}$ respectively.

5.4. Comparison between ARMA and ARMAP Forecast

The ARMAP model improves the ARMA model such that the forecast is better. The residuals of prediction with each model are used for various error analyses to determine the best model that fits the data. These error analyses may include sum of square error (SSE), sum of error (SE), mean absolute percentage error (MAPE), root mean square error and Theil’s U-Statistics. As proven inter-alia ARMA(3,0) model is found to be the best among the tested...
models. Also, the introduction of ARMAP shows a great improvement in the time series forecasting, whereby there is tremendous reduction in error analysis.

5.5. Analysis of Sum of Square Error for ARMA and ARMAP

The sums of square errors of the two methods are compared as shown in Figure 11. It can be deduced that PMA(3) for ARMA(3,0) and ARMA(4,0) have the least values with $1.87 \times 10^{-21}$ and $2.68 \times 10^{-22}$ respectively while PMA(2)+ARMA(2,0) has the highest value (255.75). Considering the principle of parsimony ARMA(3,0) will be chosen and compared with ARMA(3,0)+PMA(3). The SSE of these two models are 55.13 and $1.87 \times 10^{-21}$ for ARMA(3,0) and ARMA(3,0)+PMA(3) respectively representing a reduction of error by $6.60 \times 10^{17}$%.

5.6. Analysis of Theil’s U-Statistic for ARMA and ARMAP

The Theil’s U-statistics described in equation 18 is another analysis that describes the performance of forecasting model, usually the model with the lowest value is considered the best model. In Figure 12 ARMA(3,0) has a value 0.0061 which is higher than ARMA(3,0)+PMA(3) having $3.69 \times 10^{-14}$. However, it can be seen that ARMA(4,0) has a very slight lower value but is quite insignificant and as such it is not considered.

5.7. Analysis of Root Mean Square Error (RMSE) for ARMA and ARMAP

The model with lowest root mean square error described in equation 3.37 is considered the best model. In Figure 13 the value $1.0444 \times 10^{-10}$ ARMA(3,0)+PMA(3) for is the lowest and hence the best model. Furthermore, comparing the RMSE value of 17.2164 for ARMA(3,0) and ARMAP counterpart will indicate a difference of a value more than 1 billion.
5.8. Analysis of Mean Absolute Percentage Error (MAPE) for ARMA and ARMAP

This is another parameter used to choose the best model that fit a data. Figure 14 shows the analysis of this parameter and it can be seen that while ARMA model starts with a very high value for ARMA(1,0) it decreases up to ARMA(4,0). However, the pattern for ARMAP models starts with lower values, increases and decreased to the lowest value.

![Figure 13. Root mean square error of ARMA and ARMAP models](image1)

![Figure 14. Mean absolute percentage error of ARMA and ARMAP models](image2)

6. Conclusion

In this paper time series Box and enkins methodology is used to find a best model for forecasting mean yearly solar radiation data of 46 years, between 1971 and 2016. Results show that ARMA (3,0) model is the most parsimonious and fits the data better than other chosen models with the lowest RMSE value of 0.0004 and SSE value of 0.03885. A newly developed procedure using a hybrid method is found to improve the forecast and reduced the error between the actual and the forecasted data. ARMAP proved to be a better forecasting model where its corresponding values of RMSE, MAPE and SSE are much lower.

References
